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Music and mathematics: Exploring rhythmic structures in Indian Tala system

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Abstract
A vivid illustration of the intricate relationship that exists between mathematics and music is provided by the Indian tala system, which has rhythmic patterns that are highly improvisatory, recursive, and very organized. In this study, the Kaidas and Paltas, two of the most fundamental rhythmic compositions in Indian classical music, are the topic of analytical modeling using mathematical techniques. The complicated chains of bols, also known as mnemonic syllables, are described in a systematic manner by using formal approaches such as deep pushdown automata and state grammars. These techniques capture the structural and cross-serial interactions between the bols. The findings of this research demonstrate that computer technologies have the potential to preserve the logic of classical compositions while simultaneously precisely assessing, producing, and verifying rhythmic variations. As a result of the study's focus on the educational and creative implications of such modeling, music students, instructors, and composers now have access to a tool that allows them to better comprehend, explore, and create within existent rhythmic frameworks. Despite the fact that the mathematical modeling is extremely precise, it is not capable of capturing the nuances of performance in terms of expression, improvisation, and emotion. The way in which creative techniques and computational methods complement each other is now brought to light. In conclusion, this research contributes to the disciplines of musicology and computational music by combining the knowledge of traditional Indian music with the mathematical and computational methods that are now in use. The findings provide the platform for additional study into advanced talas, melodic integration, and interaction in music education, analysis, and composition since they give the foundation for such research.

Keywords: Indian classical music, tala system, kaida and palta, rhythmic structures, mathematical modeling, state grammar, music education

Introduction

The term "tala" originates from the root word "Taal," which indicates the usual rhythm. This word is used in the fields of music and dance. There are a number of terminologies that are used in Indian classical dance and music, and experts believe that these phrases originate from certain languages. It is important to note that the origins of the terms maa (measure) and chanda (meter) are identical in order to explain this argument. The word "tala" is supposed to have derived from the root tal, which meaning to establish. This is due to the fact that tala serves as the foundation upon which music, song, and dance are constructed. It is often thought that the name "tala" was created by combining the male dance style known as tandava and the female dance form known as lasya at some point in time. According to the Sangita Darpana, tala is the moment when the masculine and feminine elements, which are symbolised by ta (Sankara or Shiva) and la (Parvati or Shakti), come together. In Rangarnava, the "action of keeping time," which is a vital component of tala, is characterized as the sound that is produced by smacking two palms together (Krishnamurthy, 2019) [1]. One more method to comprehend the meaning of the word "tala" is to combine the action of putting the palms together and apart in order to create rhythm with the addition of a suffix to the root word "tal." One school of thought holds that those who are impacted by the ten pranas have the potential to comprehend tala and ala, which are both types of rhythm.

From Narahari Chakravarti's Bhakti Ratanakara, which is based on a statement from Ratnamala, it is said that the taakara is a sign of Kartikeya, the akara of Vishnu, and the lakara of Maruta. There is a strong theological significance linked with these gods, as shown by this connection. In spite of the fact that several historical texts provide a variety of hypotheses about the origin of tala, the core components and the manner in which they are conveyed maintain their consistency. There was a widespread perception that any attempt to break down the concept of time into smaller, more manageable chunks constituted tala. Words such as ta, dhit, thu, and nna were included in this category. In classical Bharata music, there were five distinct talas that were performed in a way known as Marga Sangitha with the purpose of bringing the gods blessings. Marga is the music that is performed for the gods, according to the common belief, while Desi is the music that people from many different regions of the globe enjoy listening to with their ears. The three subcategories of Desi that are said to have originated from Marga talas are known as Suddha, Salaga, and Sankeerna.

1. Suddhatala: Suddhatala talas are those that are uncontaminated by other talas. Two subtypes, Marga-Suddha and Desi-Suddha, make up this group. Notable among Desi-Suddha talas is Dhruva tala. The first seven talas mentioned in Somadeva's book are Prathama talas, twenty-seven are Suddha talas, and the remaining 108 are hybrids of the two types (Das & Reddy, 2018) [2].

2. Salagatala: When two different talas are mixed together, the result is called a salagatala. Two subsets of this are Desi-Salaga and Marga-Salaga. Kirti tala, a hybrid of the Vibhinna and Kokila Priya talas, is an example of Marga-Salaga, while the southern form of Dhruvarupakam is an example of Desi-Salaga.

3. Sankeernatala: The combination of many talas produces sankeernatala. Marga and Desi subtypes are also included in this group. A Marga Sankeerna Tala example would be Simhanandana, which combines elements of several talas such Caccatputa Rati, Tala Darpana, Kokilapriya, Abhanga, and Mudrika.

Basic Elements of Tala (Matra, Vibhag, Sam, Khali, etc.)

The ten elements of Tala were once referred to by the name Tala Dash Prana. These components are as follows: Kaal, Marga, Kriya, Anga, Graha, Jati, Kala, Laya, Yati, and Prastara. In the present day, the components of Tala that are known as Tali include Avartan, Matra, Laya, Bol, Vibhag, Theka, Sam, and Khali. In order to have the capacity to identify and adhere to any Tala, it is vital to have knowledge with these characteristics. The following is a catalogue of their characteristics:

Avartan: Avartan refers to the whole cycle of a certain tala. You can do it again and again. For instance, Teen Tala's Avartan

Matra: Musical time is measured in Tala units, which are called Matra. The number of matras might be the same in different talas. Consider the twelve Matras shared by ChauTala and EkTala. Having said that, they're used in distinct musical styles. For Dhrupad, use ChauTala, and for Khayal, use EkTala. While the tabla is more often used to perform EkTala, the Pakhawaj is more commonly used to play ChauTala. The shapes that they go with are well-suited to their instruments (Kumar, 2017) [3].

Laya: "Laya" is the name given to the rhythm of the song. The three main categories of

- **Vilambit laya:** Vilambit is the name of a laya set to a slow pace.
- **Madhya laya:** Madhya laya is the name given to the rhythm when it is twice as fast as Vilambit.
- **Drut laya:** Drut laya is the result of doubling the Madhya pace.

Bol: A Tala player will use the term "bol" to describe the sound they make. Dha, Dhin, tirkit, etc. are examples of mnemonic syllables. 'Patakshara' was the old moniker for them.

Theka: A Tala's Theka is its whole set of Bols, or words. Theka is the foundational framework of a Tala, and it may be further developed utilising different 'tukde' and 'tihai,' much like a 'Bandish' or musical composition.

Vibhag: Divisions called 'Vibhag' are used to disperse the theka of a Tala. Each Tala has its own unique number of divisions determined by the amount of Matras it contains. Consider TeenTala and Rupaka: the former has sixteen Matras split into four Vibhags, while the latter has seven Matras divided into three Vibha (Chandran, 2016) [4]. The Vibhags could be of the same length or different. For TeenTala, for instance, each of the four Vibhags lasts for the same amount of time, or four Matras:

Sam: The initial Matra of a Tala, which serves as the foundation upon which the whole rhythm is constructed, is called Sam. According to the Bhatkhande notation system, the symbol (Ï) is used to denote it there. As was seen in the previous example of Rupaka Tala, the 'Sam' was displayed on the first Matra. The first Matra, on the other hand, is referred to as "Khali" while it is speaking the Tala. This should be regarded an exception since the same object is often shown as "Tali," however this is not the case. This particular Matra is referred to as Sam due to the fact that, in actual practice, it serves as the point of departure for singing and playing the tabla simultaneously.

Khali: The 'Khali' is the Tala's Matra that offsets the 'Sam' and maintains a steady rhythmic cycle. In the notation used by Bhatkhande, it is represented by the number zero. When reciting a Tala verbally with hand motions, the right side of Khali's hand should be pointing upwards. There can be more than one kali.

Tali: A Tala's "Tali" is its Matra, which denotes the strike location. This is the first of several Matras performed by a Vibhag. As a result, starting with Sam, we get the first Tali, and then 2, 3, and so on, eliminating Khali.

Tala in Hindustani vs. Carnatic Traditions

The Hindustani and Carnatic tala systems are both used to organise the tempo of Indian classical music; nevertheless, they are not the same in terms of theory, terminology, notation, rhythmic units, improvisational applications, or performance approach. Both systems are used to shape the tempo of Indian classical music. Distinguishing characteristics:

Fundamental concepts

Hindustani: Tala = a framework for cyclical rhythms with a certain number of matras and portions called vibhag.

Keeping the regular cycle and improvisation in harmony is of utmost importance. A distinctive pattern of bols (syllables) provided by the theka (tabla) denotes the cycle.

Carnatic: Tala = a system constructed from anga (components) that may be joined to form talas of varying lengths; these components include laghu, drutam, and anudrutam. Integrating accurate counting procedures (kriya-based and akshara-based) with concepts codified in śrutis and organised mathematics (Śaṅkhya). Konnakol and solkattu are more formalised versions of the thekah-like pattern.

When it comes to Hindustani tala, there are less codified permutations, and the music has a long oral tradition. Additionally, it has improvisatory interaction and cyclical rhythm patterns with theka. Carnatic tala is a mathematically defined system that is made up of angas, jatis, and gatis. It is distinguished by its intricate compositional and instructional tools (konnakol, kanakku), as well as its precise akshara alignment.

Objectives

1. To use formal tools such as state grammars and deep pushdown automata to analyse and quantitatively model the rhythmic structures of the Kaidas and Paltas in the Indian tala system.
2. To investigate how mathematical modelling might be used for composition, music education, and the preservation of indigenous Indian rhythms.

Research Methodology

Using a computational-analytical research approach, this study investigates the mathematical underpinnings of rhythmic patterns in the Indian tala system, namely in the forms of Paltas and Kaidas. Namely, the work focusses on the Paltas and Kaidas. The first step was to conduct a thorough literature review on the history of tala in the Hindustani and Carnatic settings, focusing on its evolution, theoretical foundations, and structural components. For the purpose of representing and analysing the intricate patterns of Paltas and Kaidas, formal computational approaches such as state grammars and deep pushdown automata (DPDA) were used. In order to find the matra, vibhag, sam, khali, and bol arrangement, which are the fundamental building blocks of tala, the strategy that was used was to translate these elements into formal representations that could be subjected to computer analysis. DPDA models were able to represent cross-serial dependency and allowed for adjustments inside the improvisatory framework. On the other hand, state grammars were developed in order to reflect the hierarchical and recursive nature of rhythmic sequences. By systematically producing permutations and combinations of bols, which enabled the formal validation of compositions and improvisations, rhythmic variations were examined. This was done in order to study musical variations. Both quantitative and qualitative components of the music, culture, and history of Indian classical music are taken into consideration by the technique while doing the research. In the first approach, the goal is to achieve structural precision and computational repeatability via modeling. In the second approach, performance, improvisation, and expressive subtleties are taken into consideration.

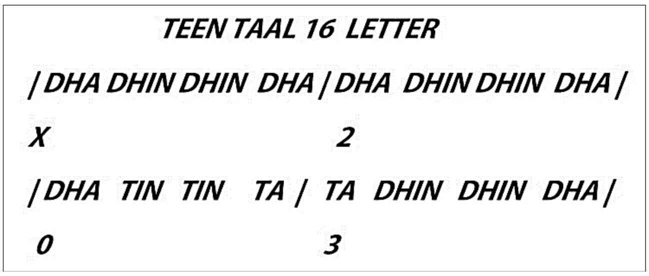
Result

Mathematical Structures in Indian Tala

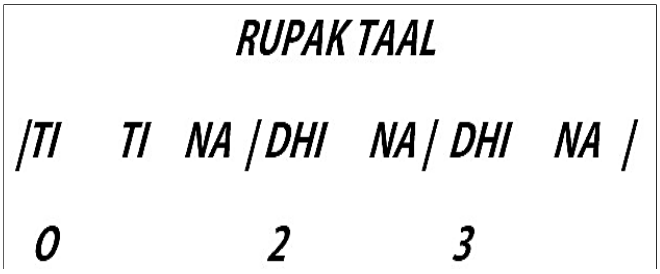
Cyclic Nature and Numerical Patterns

A full Carnatic tala has seven suladi talas. The symbols laghu, drutam, and anudrutam denote these avartana cycles' three anga. The kala (kind) and gati (pulse) of each tala are separated to improve musical performance (Randel, 2003, pp. 816-817) [5]. Each tala's "avartan" is made up of parts (anga or vibhag) that match bars or measures, but the number of beats in each section (matra or akshara) and the presence or absence of accents or pauses determines whether they are identical. Hindustani Jhoomra tals have 14 beats, although they are counted as 3+4+3+4 instead of 5+2+3+4. Spaced vibhag accents distinguish them; otherwise, two cycles of Rupak tal would sound like one cycle of Dhamar tal (Kaufmann, 1968) [6]. Most popular Hindustani tala Teental features a regularly-divisible cycle with four measures and four beats. Sam, meaning "even" or "equal" in English, is the initial beat of every tala and is always prioritized. The percussionist and soloist reach their apex here, and the soloist must play a major raga note and the piece must end in a North Indian traditional dance arrangement. The melody doesn't have to start on the first tala beat; it might be offset to accentuate the first word. Tamils call this offset talli, meaning "shift" in English. On the last beat of the previous tala cycle, some compositions begin with an anacrusis, or ateeta eduppu in Tamil.

Graphically expressing the tāla involves repetitive hand gestures called kriyas that correspond to its angas, or 'limbs', or vibhag. Any Carnatic vibhag begins with a tali ('clap') and handclapping. An "empty" (khali) vibhag is expressed by waving the dominant clapping hand (typically the right) laterally or laying the back of the hand on the base hand's palm. Hindustanis utilise these gestures to memorise and recite the tala. However, northern tala interpretations emphasize bols, individual drum strokes with names that may be recorded or spoken. A typical notation utilises 'X' for the sam and '0' for the khali, the first beat of each vibhag ("Chandrakantha Music of India," n.d.). Tala tempo (laya) is variable. Hindustani classical music ragas usually have two or three tempos: Vilambit (slow), Madhya (middle), and Drut. The pulse divisions Chauka, Vilamba, Madhyama, Drut, and Adi-drut add a slow and fast category to Carnatic music at one, two, four, eight, and sixteen strokes per beat. Northern and southern Indian classical music have theoretically developed innumerable talas from ancient times, some of which are frequent and others rare.



(A)



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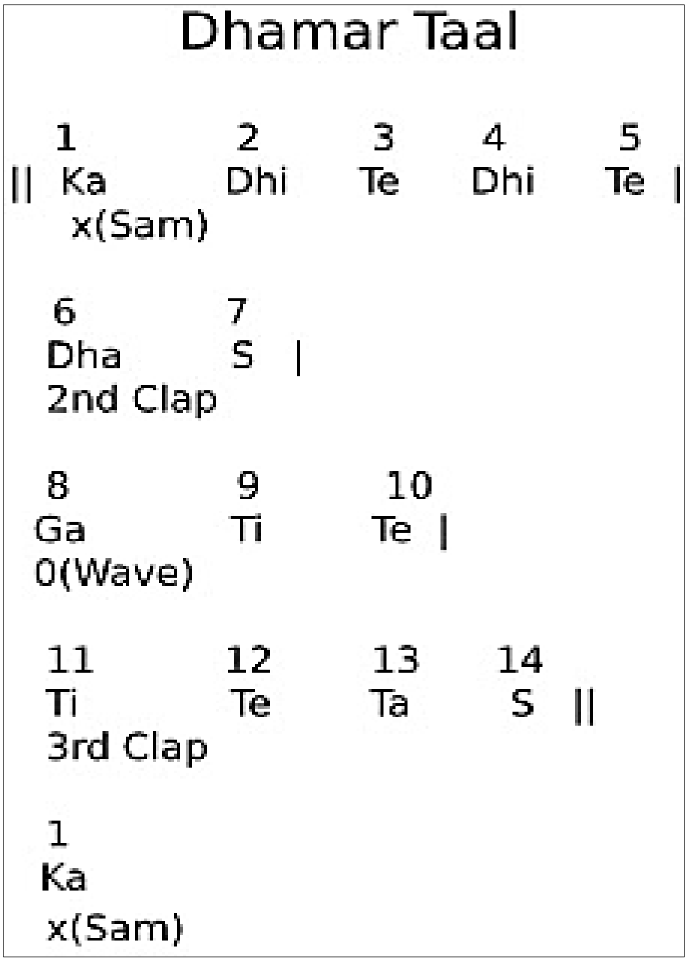

(C)

Fig 1: (A) Teentaal, (B) Rupaktaal and (C) Dhamartaal

Table 1: Typical Tabla Bols and Their Acoustic Properties

Bol (Syllable)	Sound Type	Description / Playing Technique
Ti / Te	Dry, slapping sound	Played in the black circle (*syahi*) of the dayan (right drum).
Na / Ta	Resonant tone	Played on the outer ring (keenar) of the dayan.
Tin	Resonant tone	Played on the middle ring (sur) of the dayan.
Ga	Resonant tone	Played on the middle ring (sur) of the bayan (left drum).
The	Dry slap	Played on the bayan for a sharp, percussive sound.
Dhe	Combined sound	Ga and Tin played together simultaneously.
Dha	Combined sound	Ga and Na/Ta played together simultaneously.
Trikita	Rapid sequence	Ti, Ti, Ka, Ti played quickly in succession to create a rolling sound

Mathematical Representation

Every beat location in a tala is as follows if there are n beats:

$x_t+n=x_t$

(Periodicity).

As a function iteratively returns to its starting state with each subsequent cycle, the same pattern is repeated.

Permutations, Combinations, and Rhythmic Variations

Tala is characterized in Indian music by the use of bols, which include dha, dhin, naa, ge, and tu, among others. There is a representation of X as sam in Figure 2, khaali in Figure 3, and taali in both Figures 2 and 3.

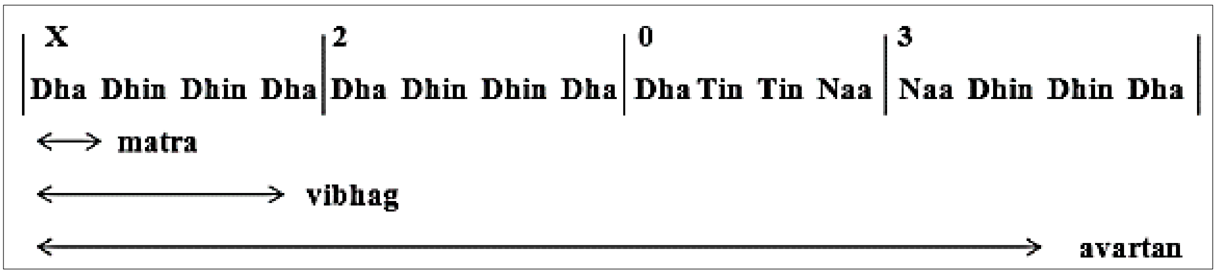

Fig 2: Theka of Teentaal showing Matra, Vibhaag, and Avartan

The structure of this Teentaal cycle is comprised of four vibhags, each of which consists of four beats (4+4+4+4). Kaida and Palta both The kaida rhythmic theme is built on the basic Talas and has an ordered series of bols. It is a kind of instrumental music. In the same way that the underlying Tala should maintain the same taali and khaali structure, the kaaida should do the same (Kaida in Indian music, n.d.). When it comes to the tabla, composition and improvisation are dependent on the two fundamental skills of kaida and palta (Bel, 1992) [8]. Kaidas are used to create paltas, which are modifications of the word. By using various improvisations on a particular kaida via the use of permutations and combinations, palta, which may be translated as "to alter" or "to reverse," are generated. It is common practice to substitute many bols of kaida for the bols that are present in kaida.

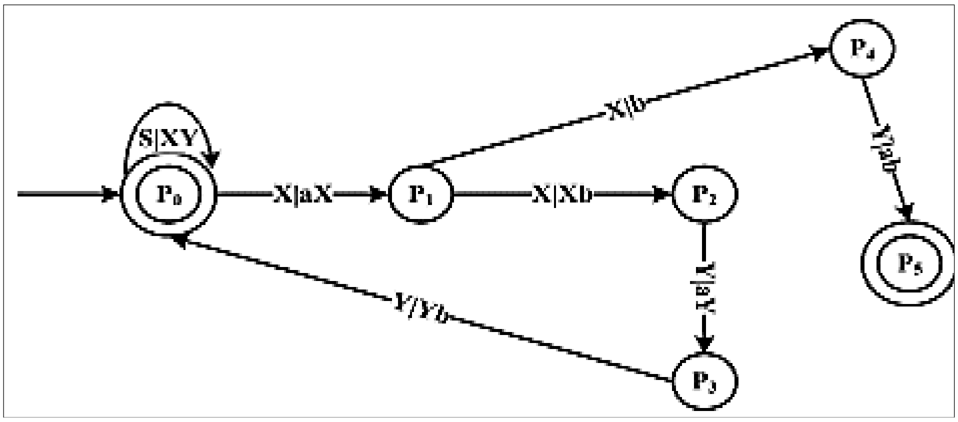

Fig 3: Anbnanb's deep pushdown automaton

Rhythmic Variations

It's possible that Tintal (teentaal), a well-known tala, might serve as an illustration of this.

The following is a list of aspects that make up Tintal:

Sixteen matra, also known as beats, comprises the duration of the tala.

The overall number of beats is sixteen, with each of the four khanda (groups of beats) consisting of four beats.

The most accentuated beat, also known as the sam, occurs on the first beat, which is represented by an x.

Two further instances of second accents, sometimes known as taali, may be found on beats five and thirteen, respectively (Bhaduri, Das, Saha, & Mazumdar, 2015) [9].

The numbers 2 and 3 are used to represent the second and third taali, which come after the sam in the sequence of numbers. When the ninth beat is reached, the letter o represents the khaali, which is a rhythm that does not have any accents.

Notation of teentaal

- Matra - 16
- Khanda - 4 (of 4-4-4-4 mattresses)
- Tali-3-1st Tali is on 1st matra (on sum)
- 2nd Tali is on 5th matra
- 3rd Tali is on 13th matra
- Khali-1-is on 9th matra

1 THAH	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Dha x	Dhin	Dhin	Dha	Dha 2	Dhin	Dhin	Dha	Dha 0	Tin	Tin	Ta	Ta 3	Dhin	Dhin	Dha
Dha x															

Fig 4: Structure of Teentaal showing 16 Matras divided into 4 Vibhaags

Temporal Division

Kala: Indicates how many matras there are that split kriya.

Laya: The interval of time between two successive kriyas. It establishes the pace.

Case Study: Mathematical Modeling of Kaidas and Paltas

Through the use of mathematical modelling of Paltas and Kaidas, it is possible to develop a methodical approach to depicting complicated patterns in Indian classical music. The correct analysis and production of these musical sequences is made possible via the use of formal approaches such as deep pushdown automata and state grammars. It is possible for this kind of modeling to capture intricate cross-serial relationships, in addition to changes in rhythm and melody (Cai & Cai, 2019, Chen, 2020, Chomsky, 1956) [10, 11, 12]. The traditional knowledge of music is combined with theoretical and computational frameworks via the development of this technique.

Representation of Indian Music Palta Using State Grammar

All of the Teentala paltas that were mentioned before had their corresponding state grammars produced.

G1 = ({S, X, Y, Dha, Te, TuNa, Ge, TiTaa, KaTa, DhaTi, DhaGe, DhinNa, GinNa, TirKit}, {q0, q1, q3, q4, q5, q6, q7, q8, q9, q10, q11, q12, q13, q14, q15, q16, q17, q18, q19, q20, q21, q22, q23, q24, q25, q26, q27, q28, q29, q30, q31, q32, q33, q34, q35, q36, q37, q38}, {Dha, Te, TuNa, Ge, TiTaa, KaTa, DhaTi, DhaGe, DhinNa, GinNa, TirKit}, P, S) let P, the Teentala paltas's state grammar, be defined in the following way:

1. (q0,s) → (q0, XY) 2. (q0, X) → (q1, DhaX) 3. (q1, Y) → (q2, DhaY)
4. (q2, X) → (q1, DhaX) 5. (q2, X) → (q3, TeX) 6. (q3, Y) → (q4, TeY)
7. (q4, X) → (q3, TeX) 8. (q4, X) → (q5, DhaX) 9. (q5, Y) → (q6, DhaY)
10. (q6, X) → (q5, DhaX) 11. (q6, X) → (q7, TuNaX) 12. (q7, Y) → (q8, TuNaY)
13. (q8, X) → (q7, TuNaX) 14. (q8, X) → (q9, TuNa) 15. (q6, X) → (q9, TuNa)
16. (q9, Y) → (q9, TuNa)

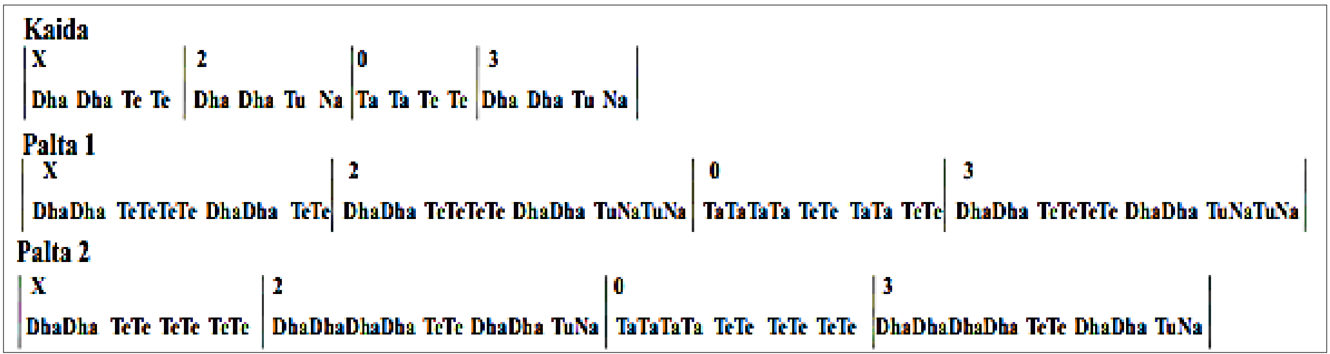

Fig 5: Palta and Kaida 1

In accordance with the information presented in Figure 4, we developed a DPDA (Figure 7) for the paltas of kaida 1. Production rules for cascade variables a, b, c, and d were also included in the final DPDA. These rules are as follows:

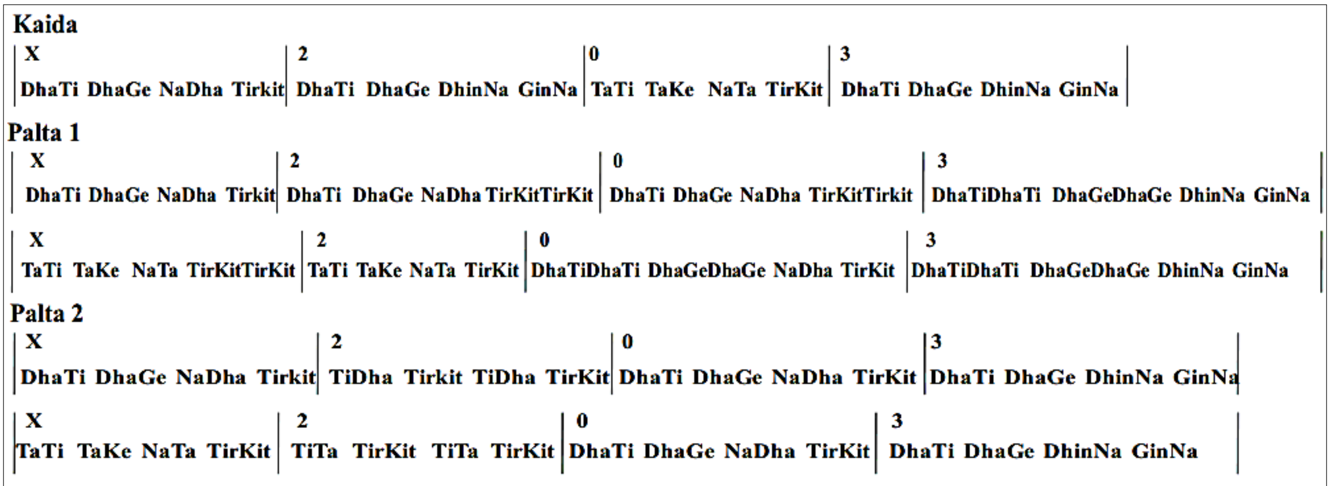

Fig 6: Palta and Kaida 2

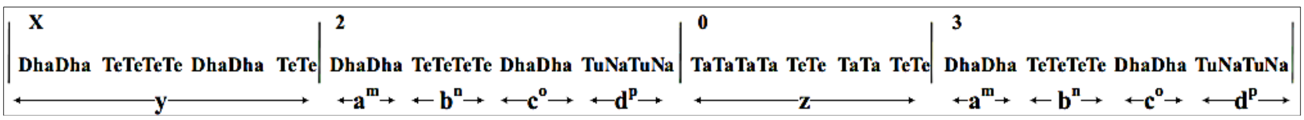

Fig 7: Language heard in the palta, or impromptu musical patterns

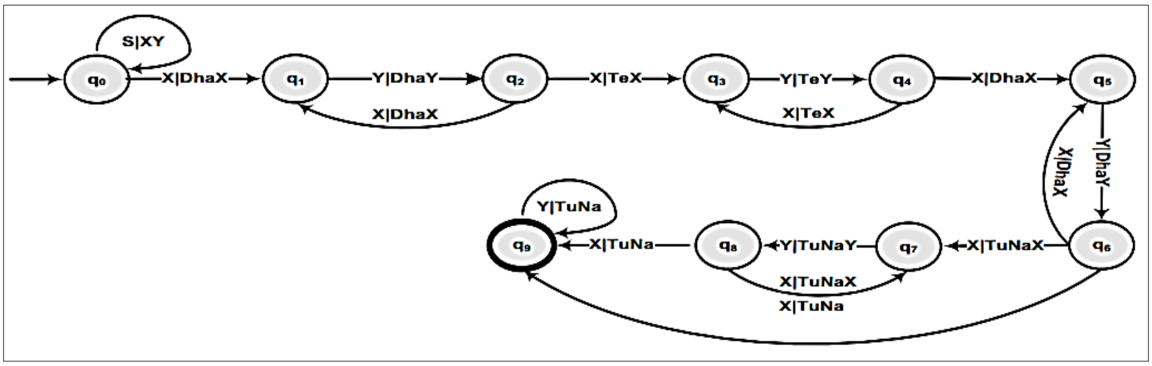

Fig 8: Paltas of Kaida 1 deep pushdown automaton

a = TeTe, GeGe, DhaDha, TiTaa, DhaTi b = TeTe, TirKit, DhaDha, DhaGe
c = GeGe, DhaDha, TuNa, DhinNa d = TeTe, TuNa,KaTa, GinNa.
All of the Teentala palta strings that have been mentioned may be produced using the production guidelines listed below.
17. (q2, X) → (q10, TirKitX) 18. (q10, Y) → (q11, TirKitY)
19. (q11, X) → (q10, TirKitX)
20. (q11, X) → (q5, DhaX) 21. (q0, X) → (q12, GeX) 22. (q12, Y) → (q13, GeY)
23. (q13, X) → (q12, GeX) 24. (q13, X) → (q12, TeX) 25. (q0, X) → (q14, TeX)
26. (q14, Y) → (q15, TeY) 27. (q15, X) → (q14, TeX) 28. (q15, X) → (q16, GeX)
29. (q16, Y) → (q17, GeY) 30. (q17, X) → (q16, GeX) 31. (q17, X) → (q18, TeX)
32. (q18, Y) → (q19, TeY) 33. (q19, X) → (q18, TeX) 34. (q19, X) → (q20, Te)
35. (q17, X) → (q20, Te) 36. (q20, Y) → (q20, Te) 37. (q0, X) → (q21, TiTaaX)
38. (q21, Y) → (q22, TiTaaY) 39. (q22, X) → (q21, TiTaaX) 40. (q22, X) → (q23, DhaX)
41. (q23, Y) → (q24, DhaY) 42. (q24, X) → (q23, DhaX)
43. (q24, X) → (q25, TuNaX)
44. (q25, Y) → (q26, TuNaY) 45. (q26, X) → (q25, TuNaX) 46. (q26, X) → (q27, KaTaX)
47. (q27, Y) → (q28, KaTaY) 48. (q28, X) → (q27, KaTaX)
49. (q28, X) → (q29, KaTa)
50. (q29, Y) → (q29, KaTa) 51. (q26, X) → (q29, KaTa) 52. (q0, X) → (q30, DhaTiX)
53. (q30, Y) → (q31, DhaTiY) 54. (q31, X) → (q30, DhaTiX)
55. (q31, X) → (q32, DhaGeX) 56. (q32, Y) → (q33, DhaGeY)
57. (q33, X) → (q32, DhaGeX) 58. (q33, X) → (q34, DhinNaX)
59. (q34, Y) → (q35, DhinNaY) 60. (q35, X) → (q34, DhinNaX)
61. (q35, X) → (q36, GinNaX) 62. (q36, Y) → (q37, GinNaY)
63. (q37, X) → (q36, GinNaX) 64. (q37, X) → (q38, GinNa) 65. (q38, Y) → (q38, GinNa)
66. (q35, X) → (q38, GinNa)

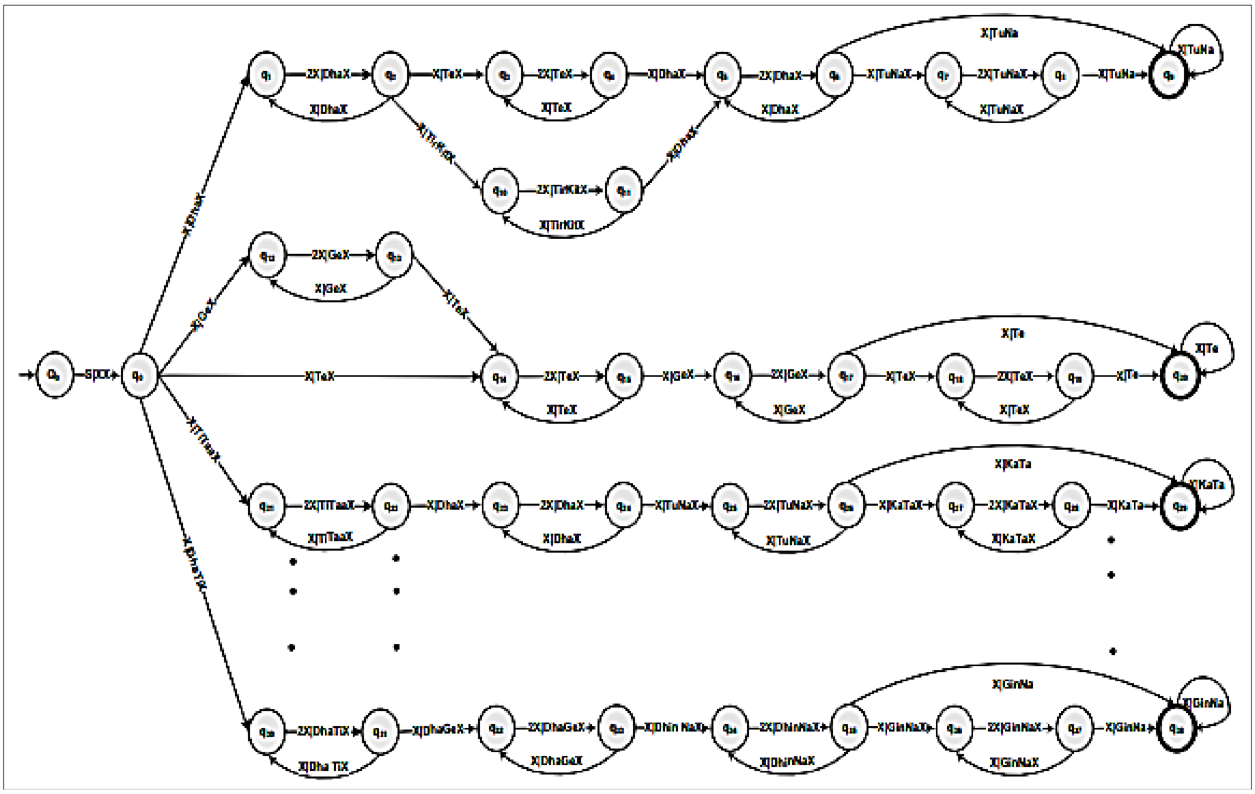

Fig 9: Automated deep pushdown for spontaneous melodies

Mathematical Modeling of Indian Music Palta Using a Deep Pushdown Automaton

A deep pushdown automaton for the palta equivalent of state grammar is shown in Figure 8. This automata is used for the purpose of describing musical strings.

M = ({Q0, q0, q1, q2, q3, q4, q5, q6, q7, q8, q9, q10, q11, q12, q13, q14, q15, q16, q17, q18, q19, q20, q21, q22, q23, q24, q25, q26, q27, q28, q29, q30, q31, q32, q33, q34, q35, q36, q37,q38}, {Dha, Te, TuNa, Ge, TiTaa, KaTa, DhaTi, DhaGe, DhinNa, GinNa, TirKit}, {X, S, #}, R, Q0, S, {q9, q20, q29, q38})

The string instrument S' = TiTaaTiTaaDhaDhaTuNaKaTa TiTaaTiTaaDhaDhaTuNaKaTa can be seen as S' = ww, where w = {TiTaamDhanTuNaoKaTap|m, n = 2;o, p = 1}.

In order to determine whether or whether this string is compatible with DPDA, it is possible to utilise the pop and expand procedures (Clayton, 1992). A representation of the DPDA may be seen in Figure 8, and the production rules (R) are as follows:

$$1Q_0S \rightarrow q_0XX \quad 1q_0X \rightarrow q_1DhaX \quad 2q_1X \rightarrow q_2DhaX$$

$$1q_2X \rightarrow q_1DhaX \quad 1q_2X \rightarrow q_3TeX \quad 2q_3X \rightarrow q_4TeX$$

$$1q_4X \rightarrow q_3TeX \quad 1q_4X \rightarrow q_5DhaX \quad 2q_5X \rightarrow q_6DhaX$$

$$1q_6X \rightarrow q_5DhaX \quad 1q_6X \rightarrow q_7TuNaX \quad 2q_7X \rightarrow q_8TuNaX$$

$$1q_8X \rightarrow q_7TuNaX \quad 1q_8X \rightarrow q_9TuNa \quad q_9X \rightarrow q_9TuNa$$

$$1q_6X \rightarrow q_9TuNa \quad 1q_2X \rightarrow q_{10}TirKitX \quad 2q_{10}X \rightarrow q_{11}TirKitX$$

$$1q_{11}X \rightarrow q_{10}TirKitX \quad 1q_0X \rightarrow q_{12}GeX \quad 2q_{12}X \rightarrow q_{13}GeX$$

$$1q_{13}X \rightarrow q_{12}GeX \quad 1q_{13}X \rightarrow q_{14}TeX \quad 1q_0X \rightarrow q_{14}TeX$$

$$2q_{14}X \rightarrow q_{15}TeX \quad 1q_{15}X \rightarrow q_{14}TeX \quad 1q_{15}X \rightarrow q_{16}GeX$$

$$2q_{16}X \rightarrow q_{17}GeX \quad 1q_{17}X \rightarrow q_{16}GeX \quad 1q_{17}X \rightarrow q_{18}TeX$$

$$2q_{18}X \rightarrow q_{19}TeX \quad 1q_{19}X \rightarrow q_{20}Te \quad 1q_{17}X \rightarrow q_{20}Te$$

$$q_{20}X \rightarrow q_{20}Te \quad 1q_0X \rightarrow q_{21}TiTaaX \quad 2q_{21}X \rightarrow q_{22}TiTaaX$$

$$1q_{22}X \rightarrow q_{21}TiTaaX \quad 1q_{22}X \rightarrow q_{23}DhaX \quad 2q_{23}X \rightarrow q_{24}DhaX$$

$$1q_{24}X \rightarrow q_{23}DhaX \quad 1q_{24}X \rightarrow q_{25}TuNaX \quad 2q_{25}X \rightarrow q_{26}TuNaX$$

$$1q_{26}X \rightarrow q_{25}TuNaX \quad 1q_{26}X \rightarrow q_{27}KaTaX \quad 2q_{27}X \rightarrow q_{28}KaTaX$$

$$1q_{28}X \rightarrow q_{27}KaTaX \quad 1q_{28}X \rightarrow q_{29}KaTa \quad q_{29}X \rightarrow q_{29}KaTa$$

$$1q_{26}X \rightarrow q_{29}KaTa \quad 1q_0X \rightarrow q_{30}DhaTiX \quad 2q_{30}X \rightarrow q_{31}DhaTiX$$

$$1q_{31}X \rightarrow q_{30}DhaTiX \quad 1q_{31}X \rightarrow q_{32}DhaGeX \quad 2q_{32}X \rightarrow q_{33}DhaGeX$$

$$1q_{33}X \rightarrow q_{32}DhaGeX \quad 1q_{33}X \rightarrow q_{34}DhinNaX \quad 2q_{34}X \rightarrow q_{35}DhinNaX$$

$$1q_{35}X \rightarrow q_{34}DinNaX \quad 1q_{35}X \rightarrow q_{36}GinNaX \quad 2q_{36}X \rightarrow q_{37}GinNaX$$

$$1q_{37}X \rightarrow q_{36}GinNaX \quad 1q_{37}X \rightarrow q_{38}GinNa \quad q_{38}X \rightarrow q_{38}GinNa$$

$$q_{35}X \rightarrow q_{38}GinNa$$

Fig 10: Representation of Teentaal Boles and Transitions in Sequential Notation

The string that was accepted as input S'=TiTaaTiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa

$$
\begin{aligned}
&(Q_0, TiTaaTiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, S\#)\\
&\Rightarrow_e (q_0, TiTaaTiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XX\#) \times [1Q_0S \rightarrow q_0XX]\\
&\Rightarrow_e (q_{21}, TiTaaTiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, TiTaaXX\#) \times [1q_0X \rightarrow q_{21}TiTaaX]\\
&\Rightarrow_p (q_{21}, TiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XX\#)\\
&\Rightarrow_e (q_{22}, TiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaX\#) \times [2q_{21}X \rightarrow q_{22}TiTaaX]\\
&\Rightarrow_e (q_{21}, TiTaaDhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, TiTaaXTiTaaX\#) \times [1q_{22}X \rightarrow q_{21}TiTaaX]\\
&\Rightarrow_p (q_{21}, DhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaX\#)\\
&\Rightarrow_e (q_{22}, DhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaX\#) \times [2q_{21}X \rightarrow q_{22}TiTaaX]\\
&\Rightarrow_e (q_{23}, DhaDhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, DhaXTiTaaTiTaaX\#) \times [1q_{22}X \rightarrow q_{23}DhaX]\\
&\Rightarrow_p (q_{23}, DhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaX\#)\\
&\Rightarrow_e (q_{24}, DhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaDhaX\#) \times [2q_{23}X \rightarrow q_{24}DhaX]\\
&\Rightarrow_e (q_{23}, DhaTuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, DhaXTiTaaTiTaaDhaX\#) \times [1q_{24}X \rightarrow q_{23}DhaX]\\
&\Rightarrow_p (q_{23}, TuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaDhaX\#)\\
&\Rightarrow_e (q_{24}, TuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaDhaDhaX\#) \times [2q_{23}X \rightarrow q_{24}DhaX]\\
&\Rightarrow_e (q_{25}, TuNaKaTaTiTaaTiTaaDhaDhaTuNaKaTa, TuNaXTiTaaTiTaaDhaDhaX\#) \times [1q_{24}X \rightarrow q_{25}TuNaX]\\
&\Rightarrow_p (q_{25}, KaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaDhaDhaX\#)\\
&\Rightarrow_e (q_{26}, KaTaTiTaaTiTaaDhaDhaTuNaKaTa, XTiTaaTiTaaDhaDhaTuNaX\#) \times [2q_{25}X \rightarrow q_{26}TuNaX]\\
&\Rightarrow_e (q_{29}, KaTaTiTaaTiTaaDhaDhaTuNaKaTa, KaTaTiTaaTiTaaDhaDhaTuNaX\#) \times [1q_{26}X \rightarrow q_{29}KaTa]\\
&\Rightarrow_p (q_{29}, TiTaaTiTaaDhaDhaTuNaKaTa, TiTaaTiTaaDhaDhaTuNaX\#)\\
&\Rightarrow_p (q_{29}, TiTaaDhaDhaTuNaKaTa, TiTaaDhaDhaTuNaX\#)\\
&\Rightarrow_p (q_{29}, DhaDhaTuNaKaTa, DhaDhaTuNaX\#)\\
&\Rightarrow_p (q_{29}, DhaTuNaKaTa, DhaTuNaX\#)\\
&\Rightarrow_p (q_{29}, TuNaKaTa, TuNaX\#) \Rightarrow_p (q_{29}, KaTa, X\#)\\
&\Rightarrow_e (q_{29}, KaTa, KaTa\#)[q_{29}X \rightarrow q_{29}KaTa] \Rightarrow_p (q_{29}, \wedge, \#)
\end{aligned}
$$

Fig 12: Stepwise Representation of Teentaal Sequence and State Transitions

An investigation of the Tala pattern in Indian music has been the subject of just a few researches that have concentrated on computational musicology. A discussion of the structure of Tala and its paltas, or variations, is presented in this paper (Diwase, 2018) [14]. The purpose of this discussion is to identify patterns in improvisational music. A language that does not take into account context is unable to recognise patterns like these because the structures of the palta of a Tala's kaida incorporate cross-serial relationships. Within the context of this paper, we expressed these patterns by using state grammar. Additionally, in order to properly portray Teentala's paltas and kaidas, we used deep pushdown automata. This research was conducted with the intention of demonstrating how the Indian tabla musical style, which is strongly reliant on improvisation, may be represented via the use of formal grammar. The kaidas and paltas patterns that are used in Indian music may be validated with the use of this procedure.

Discussion

A strong foundation for understanding the logical and structural underpinnings of rhythmic compositions such as Paltas and Kaidas may be obtained via the combination of mathematical research and musical composition. Musical phrases may be formally described as formal systems that are regulated by symmetry, recursion, and transformation rules (Dvorakova & Meduna, 2017 [15], "Importance of Indian Music," n.d., Jairazbhoy, 1995) [17]. This is a possibility. Through the use of automata theory and state grammar, we are able to comprehend the hierarchical and recursive structure of Indian rhythmic cycles, as well as the manner in which patterns evolve over the course of time. This incorporation not only sheds insight on the logic behind creative music, but it also enlightens us about the manner in which classical works maintain a balance between improvisation and structure.

Implications for Music Education and Composition

The use of mathematical models in the investigation of Paltas and Kaidas has a great deal of potential for the development of music education in the years to come. Through the process of converting abstract rhythmic concepts into computational or algorithmic forms, students are able to visualize and experiment with rhythm generation, pattern development, and variants of tala (Jurish, 2004) [18]. Composers and instructors may use these models as teaching tools to demonstrate how rhythm can change and to inspire new works that are based on classical grammar. These models would be used to highlight how rhythm can change. In addition, software that is based on these mathematical ideas gives pupils the opportunity to rehearse complex rhythmic patterns in an interactive manner. This is accomplished by merging traditional methods of education with contemporary digital methods.

Challenges and Limitations of Mathematical Modeling

There are applications for quantitative modeling; however it is not capable of capturing the essence of Indian classical music, which includes its improvisation, emotion, and expressiveness. There are numerous instances in which exact depiction fails to reflect the nuances of tempo, tone, and the interpretation of the performer. It is also exceedingly challenging to determine all of the feasible melodic and rhythmic alternatives inside a mathematical framework that is static (Kalra & Kumar, 2016a) [19]. When we place an excessive amount of importance on inflexible frameworks, we run the risk of oversimplifying the dynamic and ever-changing creative process that occurs in the field of music. Because of this, it is not sufficient to depend on mathematical approaches in order to comprehend the inner workings of Kaidas and Paltas; one must also take into consideration the cultural and artistic variables that have an effect on how they are interpreted and performed.

Conclusion

Through an investigation of the mathematical underpinnings of rhythm in the Indian tala system, this research illustrates the significant connection that exists between the two fields of study. The logic of rhythmic improvisation may be shown by formally expressing, analyzing, and producing complex patterns in Paltas and Kaidas via the use of state grammars and deep pushdown automata. It is shown in this research that mathematical models have the potential to contribute to the enhancement of music education, the preservation of classic works, and the promotion of innovative performing strategies. The expressive nuances and improvisational elements of Indian classical music, which transcend formal frameworks, demonstrate the need of striking a balance between artistic interpretation and computational modeling. In further research, it may be possible to investigate more talas, melodic features, and interactive tools with the purpose of bridging the gap between mathematics and Indian music.

References

1. Krishnamurthy S. Tala and its relationship with Abhinaya in Indian classical dance. 2019.
2. Das N, Reddy A. Tala in the performance of Kathak dance: a theoretical and practical analysis. 2018.
3. Kumar P. The structural significance of Tala in Bharatanatyam and Kathak: a cross-dance analysis. 2017.
4. Chandran K. Tala in Indian classical dance: exploring its connection to movement and music. 2016.
5. Randel D. The Harvard dictionary of music. Cambridge (MA): Harvard University Press; 2003. p. 816-817.
6. Kaufmann W. The ragas of North India. Bloomington: Indiana University Press; 1968.
7. Chandrakantha Music of India. https://chandrakantha.com
8. Bel B. Modelling improvisatory and compositional processes. Lang Des Formal Word Image Sound Synth. 1992;1(1):11-26.
9. Bhaduri S, Das O, Saha SK, Mazumdar C. Rhythm analysis of tabla signal by detecting the cyclic pattern. Innov Syst Softw Eng. 2015;11(3):187-195. https://doi.org/10.1007/s11334-015-0247-5
10. Cai L, Cai Q. Music creation and emotional recognition using neural network analysis. J Ambient Intell Human Comput. 2019;10(4):1-9. https://doi.org/10.1007/s12652-019-01614-6
11. Chen X. The application of neural network with convolution algorithm in Western music recommendation practice. J Ambient Intell Human Comput. 2020;11(2):1-10. https://doi.org/10.1007/s12652-020-01806-5
12. Chomsky N. Three models for the description of language. IRE Trans Inf Theory. 1956;2(3):113-124. https://doi.org/10.1109/TIT.1956.1056813
13. Clayton M. The rhythmic organization of North Indian classical music: Tal, lay and laykari [dissertation]. London: University of London; 1992.
14. Diwase PS. Importance of music in human life. Int J Curr Eng Sci Res. 2018;5(5):52-53.
15. Dvorakova L, Meduna A. A reduction of finitely expandable deep pushdown automata. Schedae Inf. 2017;26:61-68. https://doi.org/10.4467/20838476SI.17.005.8151
16. Importance of Indian music. https://www.shareyouressays.com/sample-essays/531-words-essay-on-the-importance-of-indian-music Tala in Indian music. https://chandrakantha.com/articles/indian_music/tala.html
17. Jairazbhoy NA. The rāgs of North Indian music: their structure and evolution. 2nd ed. Bombay: Popular Prakashan; 1995.
18. Jurish B. Music as a formal language. In: Zimmer F, editor. Finite-state automata and pure data. Das Land Steiermark: Kultur; 2004. p. 81-88.
19. Kalra N, Kumar A. Fuzzy state grammar and fuzzy deep pushdown automaton. J Intell Fuzzy Syst. 2016;31(1):249-258. https://doi.org/10.3233/IFS-162138